Lateral Forces Induced by a Misaligned Roller

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The magnitude of the lateral force induced by a roller, misaligned in its direction of travel, can be large and is not linearly proportional to the misalignment.

I. Introduction

If a metal roller which bears against a track is forced to move in a direction not perpendicular to the roller axis, a lateral force is induced. Figure 1 illustrates that the total displacement can be considered as composed of two orthogonal components, namely, one of pure rolling about the roller axis and the other of sliding parallel to the roller axis. The sliding force is $W\mu$, where W is the normal force between the roller and track and μ is the coefficient of sliding friction. The components of the sliding force parallel and perpendicular to the direction of constrained motion are respectively:

$$F_x = W\mu \sin\theta \tag{1}$$

$$F_y = -W\mu \cos\theta \tag{2}$$

For small misalignment angles, where $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, Eqs. (1) and (2) indicate that the drag force, F_x, is proportional to the misalignment, whereas the lateral force has the approximately constant value of $W\mu$. According to the concept from which Eqs. (1) and (2) are derived, the lateral force is approximately $W\mu$, because it is assumed that there has been a finite misalignment angle so that sliding has existed. Once the full value of the sliding frictional force has been established, a static frictional force can be locked in after the sliding has ceased. One can conceive of a slight misalignment being gradually reduced while the wheel is rolling. At zero misalignment the full lateral force of $W\mu$ could exist, provided the zero position had not been overshot. This concept implies that the roller and track are rigid bodies.

In reality the roller and track are elastic bodies and are deformed slightly from the induced lateral force. The effect of this deformation is to prevent the full value of the lateral force from being locked in. Within some small value of the misalignment angle, the induced side force is likely to be proportional to the misalignment angle and not proportional to the cosine of the angle as Eq. (2) indicates. A limited amount of empirical data from Ref. 1 shows the lateral force/misalignment relationship within the small angle regime.

II. Empirical Results

A few tests made at JPL using the apparatus shown in Fig. 2 gave essentially the same results as those of Ref. 1. The experiments of Ref. 1 were all made with a steel roller

0.102 m in diameter and 0.025 m wide. The track was 0.051 m wide by 0.025 m thick by 2.74 m long. Both the roller and track were 4340 steel with a yield strength of 89.6×10^7 N/m^2 (130,000 psi). The track originally had a 0.38-micron rms finish in the direction of rolling and a 1.00-micron rms finish laterally. The surface of the roller originally had 0.38-micron rms finish in both directions. Data were taken after both the roller and track had been cleaned with acetone and again after being lubricated with SAE 10W motor oil. Additional data were taken after both the track and roller had been sandblasted to a surface finish of 4.50 microns rms. Subsequently the track and roller were rusted to simulate outdoor conditions. The force between the roller and track ranged from 1800 to 15100 N (400 to 3400 lb). This corresponds to a nominal Hertz stress of 22×10^7 to 64×10^7 N/m^2 (31,900 to 93,000 psi).

Some of the results of the Ref. 1 experiments are shown in Figs. 3, 4 and 5. In these figures the lateral force coefficient is the ratio between the lateral and normal forces. Figures 4 and 5 indicate that there are no significant differences between force coefficients produced by various surface conditions. Apparently the viscosity of the SAE 10W oil was insufficient to provide a film at these values of Hertz stress. Figures 3, 4 and 5 each show an almost linear relationship between lateral force coefficient and misalignment angles up to about 10 arc min, and a fairly constant value of lateral force coefficient for misalignments greater than 30 arc min.

III. Consequences of the Induced Lateral Forces

A method of obtaining perfect alignment of a roller in its direction of motion is through the use of caster, which means that misalignments are reduced to zero as the wheel advances. However, the reversal of motion of a wheel with caster causes a misalignment to increase. For those cases where the wheel must move in both directions, the wheel or roller design should provide for: (1) achieving and maintaining an alignment within certain small amounts in order to limit the lateral force to certain small values, or (2) accepting and coping with the large lateral force which will exist if the misalignment exceeds a few arc min of angle. Figures 3, 4 and 5 give some insight to the problem. It is emphasized that these figures are based upon a limited amount of empirical data and may not quantitatively apply to a specific application. It would appear, however, that the maximum lateral force coefficient is approximately the same as the sliding coefficient of friction when the wheel is sliding parallel to its axis of rotation. If this sliding friction coefficient could be determined for the particular application, it is believed that it would be approximately the same as the maximum induced lateral force coefficient.

Usually it is desirable that the load intensity between the wheel and track be as nearly uniform as possible. For some wheel suspension configurations the induced lateral force requires an interface moment for static equilibrium; thus the load intensity across the width of the wheel is nonuniform, the maximum intensity being much above the average. This same suspension configuration may well accommodate for a warped track. Therefore, it is most important to consider carefully the suspension design of a wheel or roller when induced side forces exist and caster cannot be employed.

IV. Examples of Wheel Suspensior Designs

Figure 6(a) shows a wheel without caster mounted so as to allow the wheel frame to pivot about point A in order to permit the wheel to remain flat against the track even if there is a small angular displacement between the track and base structure. Figure 6(b) shows a free body diagram of the wheel and frame. The forces V and S are, respectively, the total normal force between wheel and track and the induced lateral force. For instances of small angular displacement between the track and base structure, where the configuration remains essentially constant, a summation of moments about the wheel-track origin yields the following expression for the required equilibrating interface moment, M_i:

$$M_i = aS \mp VR\mu_B \tag{3}$$

where a and R are shown in Fig. 6b, μ_B is the coefficient of friction of the pivot. Setting $S = \mu V$ where μ is the coefficient of sliding friction between the wheel and track, Eq. (3) becomes:

$$\frac{M_i}{V} = a\mu \mp R\mu_B \tag{4}$$

The $\pm$ sign in Eqns. (3) and (4) accounts for the direction of motion of the pivot.

For the case where there is no relative angular displacement between the track and base structure, it is possible that the two terms on the right side of Eq. (4) always have opposite signs. In this case the value of M_i could be very low. If instead of an intentional pivot at A the wheel frame is solidly attached to the base frame, the effective value of $R\mu_B$ could be large enough to cancel the $a\mu$ term. For such a case, however, the real value of M_i will depend upon the elastic stiffnesses of the various parts of the structure. A certain 30-m-diam antenna is known to have a very precise azimuth track. Its wheels are mounted so that there is no intentional pivot at point A. The wheel and track wear after four years of operation has been both small and uniform.

For the case where there is an intentional pivot at point A the two terms of the right side of Eq. (4) will at times have the same signs and the maximum absolute value of M_i/V will be:

$$\left| \frac{M_i}{V} \right|_{max} = \left| a\mu + R\mu_B \right| \tag{5}$$

A certain 100-m antenna had wheels suspended such that $R\mu_B$ was effectively very small but the ratio a/L was 6.58. After four years of operation there was enough nonuniform wear to necessitate a remachining of the wheels and track. These wheels and track are nominally identical to those mentioned in the previous example and the nominal load per wheel is the same.

Since the wheel and track coefficient of sliding friction μ is fixed, only a, R and μ_B are available for reduction in magnitude. If the center of R is placed at the wheel track origin, "a" becomes zero, but R becomes large if configured as shown by the dashed line of Fig. 6a.

A suspension system which effectively places point A at the wheel-track origin and effectively make R small is shown in Fig. 7. It has the advantage of allowing the wheel or roller to accommodate to a slightly warped track and does not require a larger equilibrating interface moment in the presence of an induced lateral force. A complete analysis of this design will appear in a separate report.

Reference

1. Cross, H. A., and Talbert, S. G., *Experimental Determination of Lateral Forces Developed by a Misaligned Steel Roller on a Steel Rail*, Machine Design Division of ASME, Oct. 2, 1963.

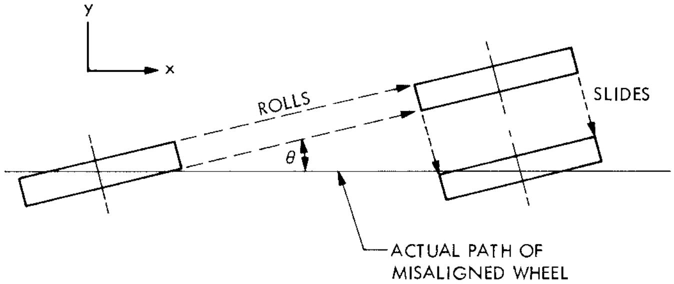

Fig. 1. Model of induced lateral force

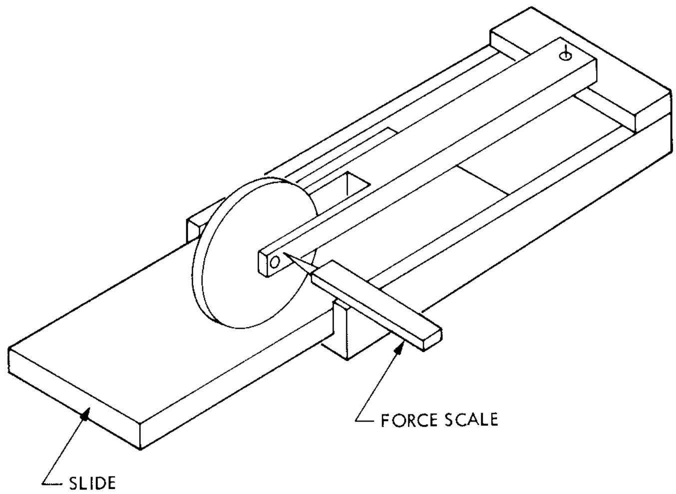

Fig. 2. Test apparatus used at JPL

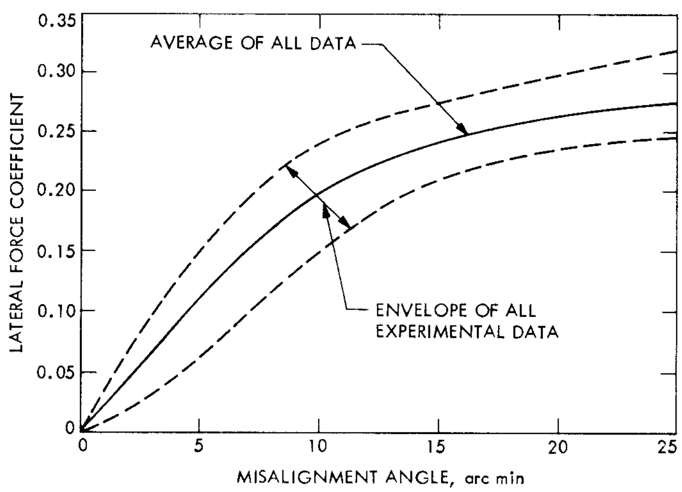

Fig. 3. Lateral force coefficient vs small misalignment

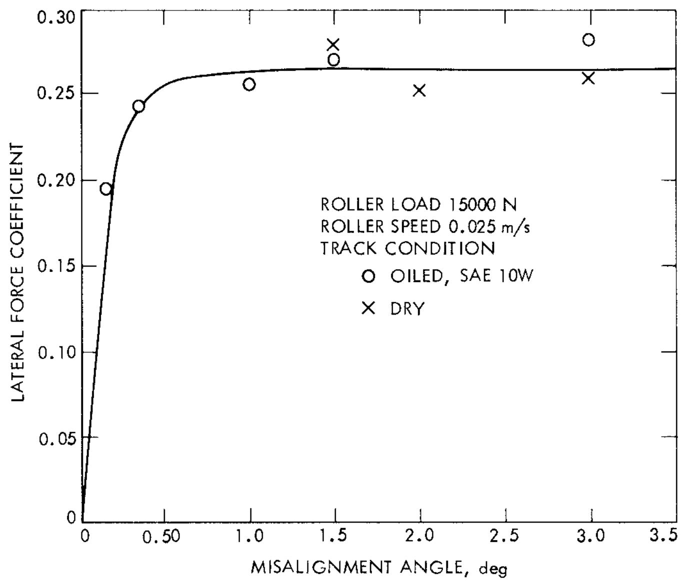

Fig. 4. Lateral force coefficient vs large misalignment

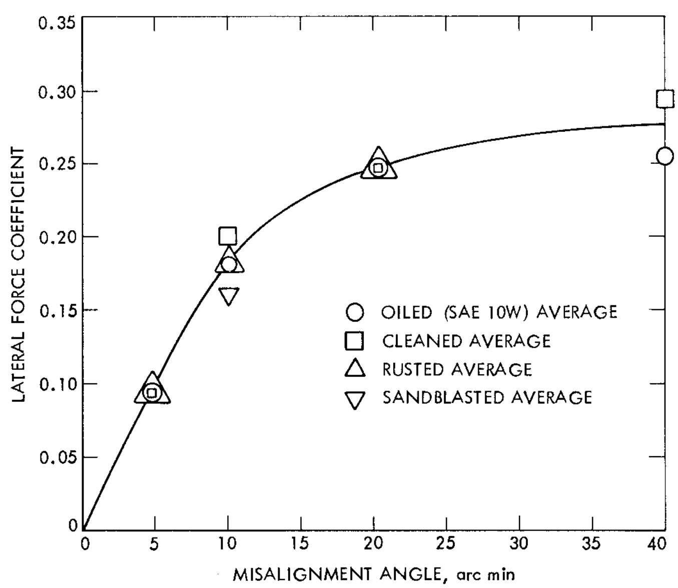

Fig. 5. Effect of surface condition on force coefficient

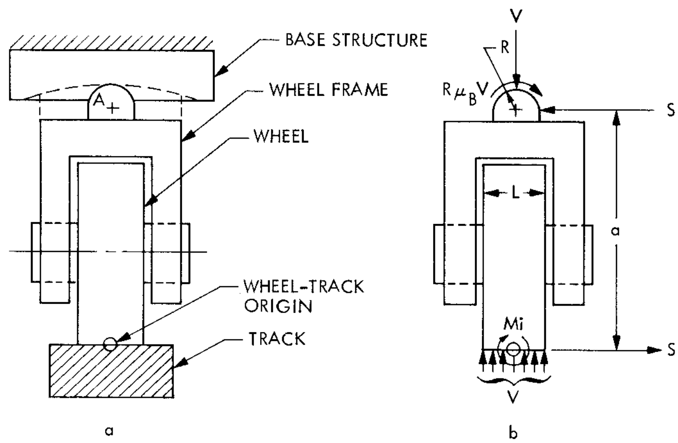

Fig. 6. Wheel suspension

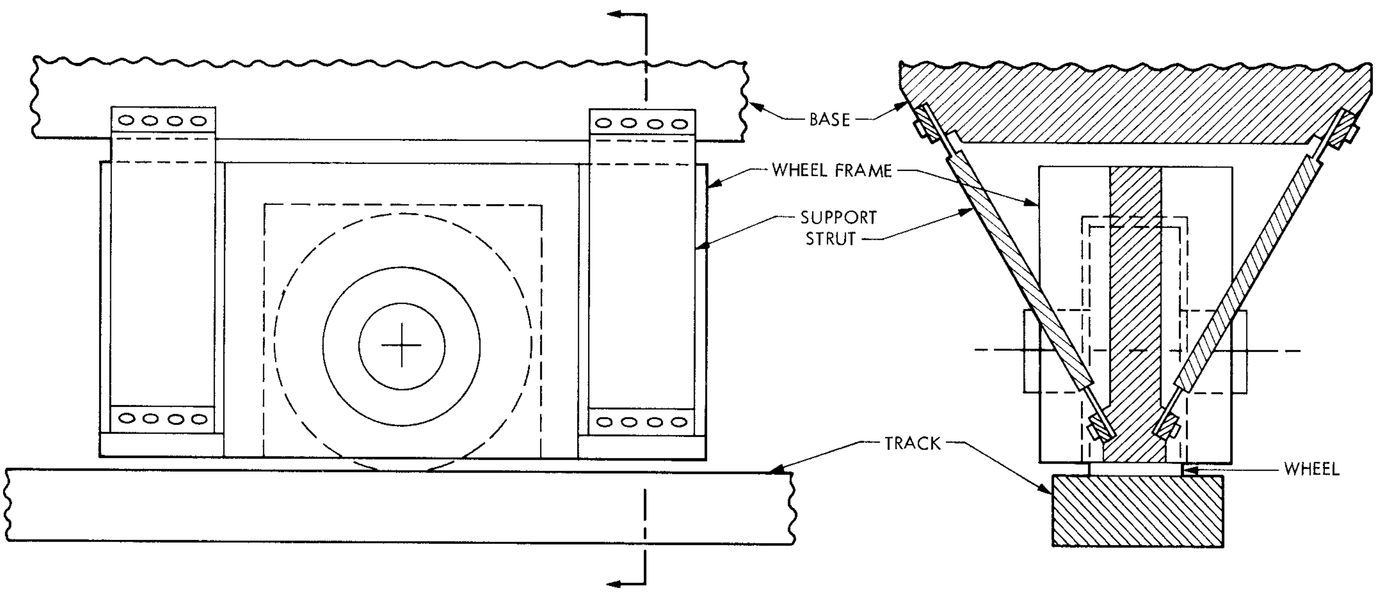

Fig. 7. Wheel suspension with small interface moment